# ARITHMETIC.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

73. Proposed by NELSON S. RORAY, South Jersey Institute, Bridgeton, New Jersey.

A man owes me \$100 due in 2 years, and I owe him \$200 due in 4 years. When can I pay him \$100 to settle the account equitably, money being worth 6%, and the interest to draw interest until the time of settlement?

Solution by FREDERIC R. HONEY, New Haven, Connecticut.

One dollar placed at 6% compound interest, in two years will amount to $1.06^2=\$1.1236$. Therefore the present value of \$100.00 due in 2 years is $\$100.00\div1.1236=\$89.00$ *very nearly.*

One dollar placed at 6% compound interest in four years will amount to $1.06^4=\$1.2625$. Therefore the present value of \$200.00 due in 4 years is $\$200.00\div1.2625=\$158.416$.

Therefore the difference between \$158.416 and $\$89.00=\$69.416$, is the amount of my debt to $A$ at the present time.

Since \$1.00 placed at 6% compound interest in 6 years will amount to $1.06^6=\$1.4185$, \$69.416 at the same rate will, in 6 years, amount to $69.416\times 1.4185=\$98.4666$.

And since the simple interest on one dollar for 1 year is \$0.06, the simple interest on \$98.4666 is $98.4666\times0.06=\$5.908$ for one year. Therefore the interest $\$100.00-\$98.4666=\$1.5334$ will accrue in $1.5334\div5.908=0.2594$ years.

And $6.+0.2594=6.2594$ the number of years hence when \$100.00 should be paid, in order to settle the account equitably.

J. M. Bandy sent solutions of Nos. 71 and 72 too late for credit in February number.

# GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

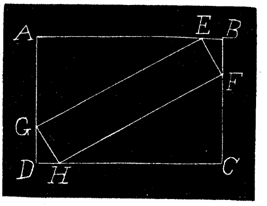
## SOLUTIONS OF PROBLEMS.

67. Proposed by F. M. PRIEST, St. Louis, Mo.

Required: The length of a piece of carpet that is a yard wide with square ends, that can be placed diagonally in a room 40 feet long and 30 feet wide, the corners of the carpet just touching the walls of the room.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas; P. S. BERG, Larimore, North Dakota; J. SCHEFFER, A. M., Hagerstown, Maryland; J. M. COLAW, A. M., Monterey, Virginia; R. H. WAGONER, Westerville, Ohio; J. F. YOTHERS, Westerville, Ohio; J. T. FAIRCHILD, Crawfis College, Ohio; CHAS. C. CROSS, Laytonsville, Maryland; and O. S. WESTCOST, Chicago, Illinois.

Let $AB=40=a$, $BC=30=b$, $EF=3=c$, $BF=x$, $BE=y$.




$\therefore\ x^2+y^2=c^2$ ........................(1).

From the triangles $CHF$ and $BEF$ we get $HC:CF=BF:BE$ or $a-y:b-x=x:y$.

$\therefore\ ay-y^2=bx-x^2$ ....................(2).

(1) in (2) gives $bx-x^2=a\sqrt{c^2-x^2}-c^2+x^2$.

$\therefore\ 4x^4-4bx^3+(a^2+b^2-4c^2)x^2+2bc^2x+c^4-a^2c^2=0$.

$\therefore\ 4x^4-120x^3+2464x^2+540x-14319=0$.

$\therefore\ x=2.43372+$, $y=1.75414+$.

$HG=\{(a-y)^2+(b-x)^2\}^{\frac{1}{2}}=47.14494+$.

II. Solution by COOPER D. SCHMITT, Professor of Mathematics, University of Tennessee, Nashville, Tennessee; W. H. HARVEY, Portland, Maine; and B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

In the figure used above, let $AB=DC=a=40$ feet, the length of the room.

$AD=BC=b=30$ feet, the width of the room.

$EF=GH=c=3$ feet, the width of the carpet.

Let $x=HF=GE$, the length of the carpet, and the angle $CFH$=the angle $HGD=\theta$.

Then $x\sin\theta=CF$, $c\sin\theta=DH$, $x\cos\theta=HC$, and $c\cos\theta=DG$.

$\therefore\ c\sin\theta+x\cos\theta=DC=a$ ..................................(1),

and $x\sin\theta+c\cos\theta=BC=b$ ........................................(2).

Multiplying (1) by (2), and collecting, we get

$cx(\sin^2\theta+\cos^2\theta)+(x^2+c^2)\sin\theta\cos\theta=ab$, or

$$cx+(x^2+c^2)\sin\theta\cos\theta=ab \text{ ..............(3).}$$

Squaring (1) and (2) and adding the results, we get

$$c^2+x^2+4cx\sin\theta\cos\theta=a^2+b^2 \text{ ..................(4).}$$

From (3), $\sin\theta\cos\theta=(ab-cx)/(x^2+c^2)$. Substituting this value of $\sin\theta\cos\theta$ in (4) and reducing, we get,

$$x^4-(a^2+b^2+2c^2)x^2+4abcx-c^2(a^2+b^2-c^2)=0\ \ldots\ldots\ldots\ldots(5).$$

Restoring numbers in (5), we have

$$x^4-2518x^2+14400x-22419=0.$$

Solving this equation by Horner's Method, we find $x=47.145$ feet, nearly.

# CALCULUS.

Conducted by J. M. COLAW, Monterey, Virginia. All contributions to this department should be sent to him.

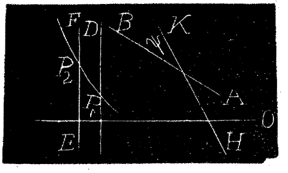
## SOLUTIONS OF PROBLEMS.

**58.** Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

A line passes through a fixed point and rotates uniformly about this point. Another line passes through a point which moves uniformly along the arc of a given curve and rotates uniformly about this point. Develop a method for finding the locus of intersection of these two lines. Apply to case of circle and straight line.

II. Solution by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts.

Let $O$ be the origin, $P_3$ the fixed point, its coördinates being $(r_3,\ \theta_3)$, and let $AB$ be a given position of line through $P_3$. Let $P_1(r_1,\ \theta_1)$ be position of point on curve and $CD$ the line through it, both corresponding to the position $AB$ of other line. Also let $HK$ be position of $AB$ revolved through an $\angle\psi$, and let $P_2(r_2,\ \theta_2)$ and $EF$ be the corresponding position of $P_1$ and $CD$.




Let $r=f(\theta)$ be equation to curve $P_1P_2$. Let $\eta$=the angle made by $AB$, and $\eta_1$ the one made by $CD$ with a polar axis. Let $a$=angular rate of revolution of $AB$, and $na$ of $CD$.

$\therefore\ \angle$ between $CD$ and $EF=n\psi$.

Let $b$=linear rate of movement of $P_1$. Then $\psi/a=P_1P_2/b$ ......... (1).

Equation to $KH$ is $r=[r_3\sin(\eta+\psi-\theta_3)]/\sin(\eta+\psi-\theta)$ .............(2).

Equation to $EF$ is $r=(r_2\sin(\eta_1+n\psi-\theta_2)]/\sin(\eta_1+n\psi-\theta)$..........(3).

By integration,